

Anvendelse av metoder for datareduksjon på topografiske LIDAR data

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Trond Nordvik and Terje Midtbø: Application of Data Reduction Methods in Dynamic TIN Models of Topographic LIDAR Data

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Comparisons of five data reduction methods associated with dynamic TIN models were conducted. All methods were applied to real world Light Detection and Ranging (LIDAR) topographic data. Data reduction was based on point selection by thresholding in dynamic Delaunay triangulation combined with random point selection. The triangulation criteria used include both Delaunay triangulation and hybrids of Delaunay and data dependent triangulation. The performance of various reduction methods was evaluated by means of surface area, volume, RMS of vertical errors and maximum vertical errors. All methods were evaluated using five levels of reduction; 10%, 5%, 2.5%, 1% and 0.5% of full datasets.

Key words: Data reduction, TIN, Digital Terrain Modelling, LIDAR

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1. Introduction

Although the processing capacity of desktop computers is continually increasing, the large data volumes generated by some data collection technologies such as Light Detection and Ranging (LIDAR), pose a challenge when it comes to processing, for example spatial analyses and visualization. Efficient data reduction methods are required. The objective of data reduction is to reduce the amount of data and at the same time preserve as much useful information as possible. A wide range of data reduction methods associated with polygon meshes are reported in the literature. Two main approaches for data reduction can be identified. The first approach starts with a full model containing all data points and excludes the less significant points which are deemed redundant at the chosen level of decimation. The second approach starts with a coarse mesh of some key points and successively adds data points which are deemed significant.

Examples of the first approach include the surface simplification method suggested by Schröder and Roßbach [1]. This method ini-

tially generates a high resolution TIN representing the given surface and then repeatedly reduces the number of points in this mesh. The criterion used to assess the significance of a point calculates a measure of the roughness of the terrain at that point. When the roughness value of a point indicates that it can be removed from the mesh without affecting the overall representation significantly, the area around the removed point is retriangulated. The simplification algorithm proposed by Garland and Heckbert [2] is based on contraction of vertex pairs in a full model. Demaret et al [3] also used an approach based on successive removal of points from a full model.

Examples of the second approach include adaptive triangular mesh (ATM) filtering as proposed by Heller [4]. This approach is the conceptual equivalent of the Douglas-Peucker (DP) approach for lines. The triangulation starts with a few initial points. Depending on the vertical distance to the current triangular mesh, new points are added to the triangulation until no relevant points remain. Bottelier et al [5] also use a combination of a

«DP»-like algorithm and thresholding in a dynamic TIN model for near-real-time decimation of high density echo sounder data. This latter approach is much more favourable in terms of the processing demand. The methods evaluated in this paper utilize this approach.

The performance of data reduction methods can be evaluated in terms of criteria such as approximation errors and processing time. Garland and Heckbert [2] used error quadrics for the triangle planes joined at each vertex as a measure of approximation error. Demaret et al [3] used CPU time and maximum approximation error as evaluation criteria. Anderson et al [6] compared the mean and standard deviation of full datasets with those of reduced datasets. Choice of criteria is highly application dependent. In visualization applications, for example, one of the most important properties to preserve is the surface normal vectors Diebel et al [7]. In this study, the class of polygonal meshes dealt with is limited to Triangular Irregular Networks (TINs). The TIN model represents a surface as a set of contiguous, non-overlapping, irregularly shaped triangular facets. A TIN model can be regarded as both a terrain model and a data structure. It is a model because the space-filling triangular planar facets determine a value for the surface everywhere, but it also has a specific structure in which the points, triangles and topologies are stored [8]. The most common types of TIN models are Delaunay triangulations but data dependent triangulations are also well established.

To discuss the advantages and disadvantages of TIN models it is useful to introduce grid models or digital elevation models (DEMs) as a reference. A DEM consists of a regular mesh of cells that are usually square. The TIN and DEM approaches are by far the most common types of terrain models. In general, TIN models are more complex with respect to algorithms and data structures and may require more memory storage than DEM models, depending on the implementation. These are the main disadvantages of TIN models. On the other hand, TIN readily allows for variable resolution depending on topography. TIN can represent morphology

directly and better than grid-based models, since these models allow vertices to be placed at arbitrary positions, and arbitrary polygons to be embedded among their edges [9]. By using the sampled points directly and avoiding the «gridding» process, one avoids introducing errors associated with the «gridding» process and a more precise representation of a topographic surface is produced. In this respect one could say that a TIN model is a model that fits the data and not vice versa. Moreover, many graphics systems are optimized for rendering triangles.

Comparison of data reduction methods in TIN models applied to real world topographic data has not received much attention in the literature. However, some research has been done including Demaret et al [3]. The objective of the research reported in this paper is to quantify the performance of some data reduction methods using real world data. Five data reduction methods (random subset and two types of threshold in dynamic Delaunay triangulation, each combined with a final data dependent edge swap pass) have been applied to LIDAR topographic data. All of the data reduction methods evaluated in this study provide a subset of the full dataset by selection of points.

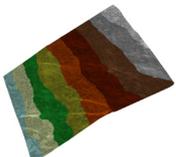
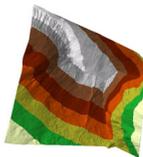
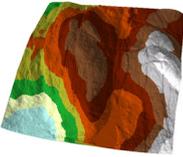
1.1 Description of Data

Analyses in this study are based on data collected with airborne LIDAR. For a general description of the LIDAR technology, see for example Baltasvis [10] and the references therein. Technical specifications of the particular LIDAR system «Optech Airborne Laser Terrain Mapper (ALTM3100C)» used for data collection of this study can be obtained from Optech [11]. The LIDAR data comprise point measurements: x-, y- and z-coordinates and the intensity of the reflected signal. The background for this scan campaign is landslide hazard assessment. The area covered in this scan is along the fjord system of inner Storfjorden in Møre and Romsdal County, Norway. It covers nearly 600 square km, mostly steep hillsides from fjord to mountain. From this total area, 32 datasets (each covering a square area of 1km by 1km) were selected as test datasets for this study. Data were pre-processed by the contractor and

thus assumed to contain only bald earth measurement with no gross errors present. The point density of the test datasets ranges from 0.41 to 1.53 points per square meter, with an average of 0.82 points per square meter. In some of the test areas the point density varies somewhat, particularly where two or more adjacent strips overlap. The elevation range from the highest to the lowest

point within a single dataset varies from about 250 meters to about 1200 meters. To assess the performance of the data reduction methods on different topographies, the 32 test datasets were sorted by discretion into four categories: «hilly», «hillside», «ridge/peak» and «valley/pit». See Table 1 for a description of the categories and an example of each.

Table 1. Terrain categories used in the study.

Category	«Hillside»	«Ridge/Peak»	«Valley/Pit»	«Hilly»
Description	planar slopes, when microtopography is disregarded	concave terrain surfaces, when microtopography is disregarded	convex terrain surfaces, when microtopography is disregarded	broken and undulating terrain that does not fit into any of the other categories
Representative dataset				

2 Implementation and Calculations

2.1 Experiment

Five data reduction methods were applied to 32 test datasets. These were called RD, PD, PH, VD and VH and are described in Table 2. Each dataset covers a square area of 1 km x 1 km, with an average of 820.000 points per km². The following criteria were selected to evaluate the approximation errors of the various methods: surface area, volume, root mean square (RMS) of vertical residuals, mean of vertical residuals and maximum

vertical residuals. Vertical residuals were calculated as the difference between the measured values of the control points and the corresponding interpolated values from the TIN surfaces. Since there were no accurate control measurements available, the omitted points were used as control points. Surface area and volume calculations obtained from the reduced datasets were compared with those obtained from the corresponding full datasets. All methods were evaluated for five levels of reduction: 10%, 5%, 2.5%, 1% and 0.5% of full dataset.

Table 2. Specification of data reduction methods. A hybrid triangulation criterion means that the final Delaunay triangulation was modified by a data dependent edge swap pass. The *em-braced texts specify the data dependent swap criteria.*

Name	Point selection criterion	Triangulation criterion
RD	Random	Delaunay
PD	Perpendicular threshold	Delaunay
PH	Perpendicular threshold	Hybrid of Delaunay and data dep. (minimize vertical RMS)
VD	Vertical threshold	Delaunay
VH	Vertical threshold	Hybrid of Delaunay and data dep. (minimize max vertical error)

2.2 Description of TIN Data Structures and Algorithms

A dynamic Delaunay triangulation algorithm was used as a basis algorithm in this investigation. The dynamic algorithm starts with a coarse mesh of key points and allows for addition and removal of points during the triangulation process. See Midtbø [12] for a thorough description of data structures and algorithms for dynamic Delaunay triangulations. This basis algorithm was extended with functionality that would accommodate

the comparison study. Microsoft Visual Studio and C++ were used for the implementation. A twin edge data structure, proposed by Heller [4] provided the backbone of the triangulation. All information about the triangles is implicitly stored in the edges and consequently no triangle table is required. However, to facilitate the triangle-by-triangle calculations needed for this study, a triangle table was considered to be a convenient extension. The topology of the network was maintained by pointers between edges (Figure 1).

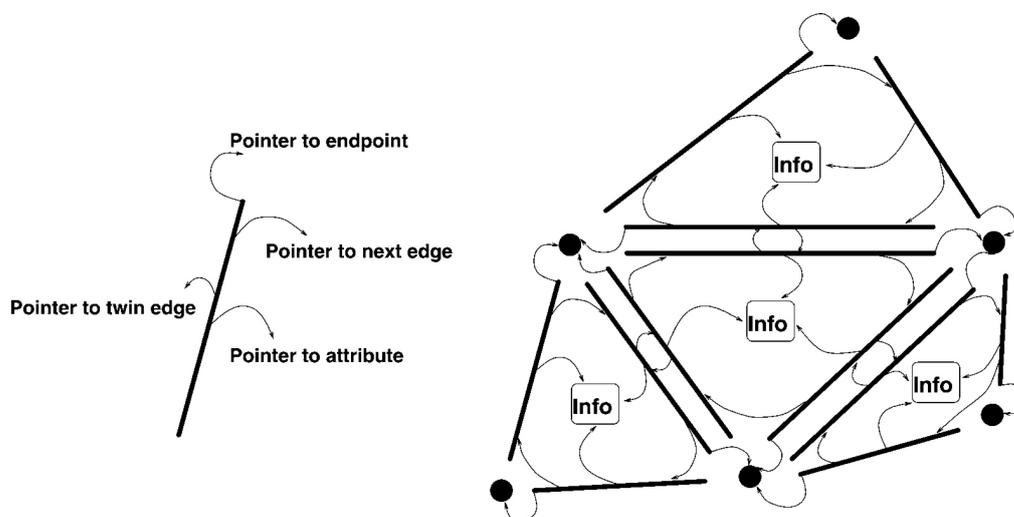


Figure 1. Twin-edge data structure (figure from Midtbø [12])

Initially, a triangulation of the corner points of a circumscribing rectangle of the dataset was chosen as an approximation mesh. The height of these initial auxiliary vertices was arbitrarily set to -300 meters, which resulted in the selection of the point with the highest elevation as the first inserted point in each of the two initial triangles. When a new point was inserted into the network, the enclosing triangle of the current point was split into three new triangles and the data structure updated accordingly, see Figure 2.

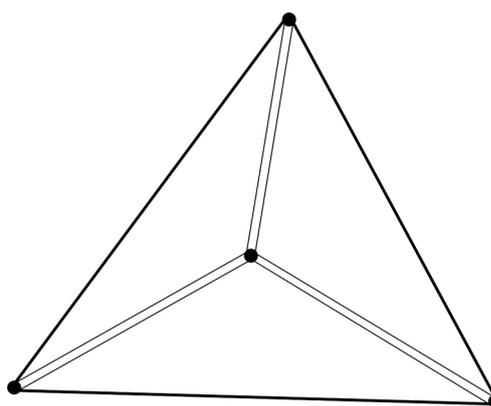


Figure 2. Insertion of a new point. The centre point was inserted and 6 new edges added to the network (figure from Midtbø [12])

In order to maintain the Delaunay criterion when a new point was inserted, a recursive edge swap procedure was applied if the criterion was not met. Extension to data depend-

ent edge swap is a minor adaptation of the existing Delaunay triangulation framework. The following pseudo code outlines the idea:

```

Pseudocode for data dependent edge swap
for each edge[i] element of Delaunay triangulation
  if( isSwapable(edge[i]))
    if( getCostCur(edge[i]) > getCostAlt(edge[i]))
      swapEdge(edge[i])
    
```

The `isSwapable` function is needed since quadrangles that are not convex will not gen-

erate consistent triangles when the diagonal is interchanged, see Figure 3.

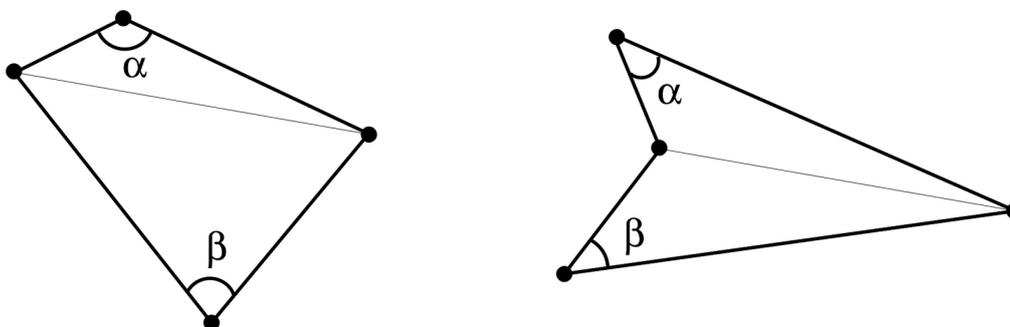


Figure 3. Quadrangles. The left quadrangle is convex and the diagonal is swappable. The right quadrangle is concave and the diagonal is not swappable (figure from Midtbø [12])

The two functions `getCostCur` and `getCostAlt` calculate the costs associated with the two alternative configurations of the diagonal. The cost function in the case of method VH is maximum vertical residual. This will preserve the most useful property of the vertical distance method, that is, to control the maximum vertical residual. The cost function in the case of method PH is RMS. The `swapEdge` function performs the interchange of diagonal and updates the data structures.

2.3 Selection of Data Point

For the random selection criteria each data point was included in the TIN model with a probability p . For the vertical distance criterion, the point with the greatest vertical distance from the existing TIN surface was in-

serted into the triangulation, given that the distance was larger than a specified threshold (Figure 4). The procedure for the perpendicular criterion was identical to the vertical distance criterion, except that the vertical distance from the existing triangle plane was replaced by the perpendicular distance. Selecting the point with the greatest perpendicular distance to the existing triangle plane corresponds to selecting the point having the greatest influence on volume for the splitting of this triangle. The point with the greatest perpendicular distance forms the apex of an imaginary tetrahedron with the existing triangle plane as the base. Thus, this is the point that will have the largest impact on volume. Both the vertical and the perpendicular distance criteria were combined with a final edge swap pass.

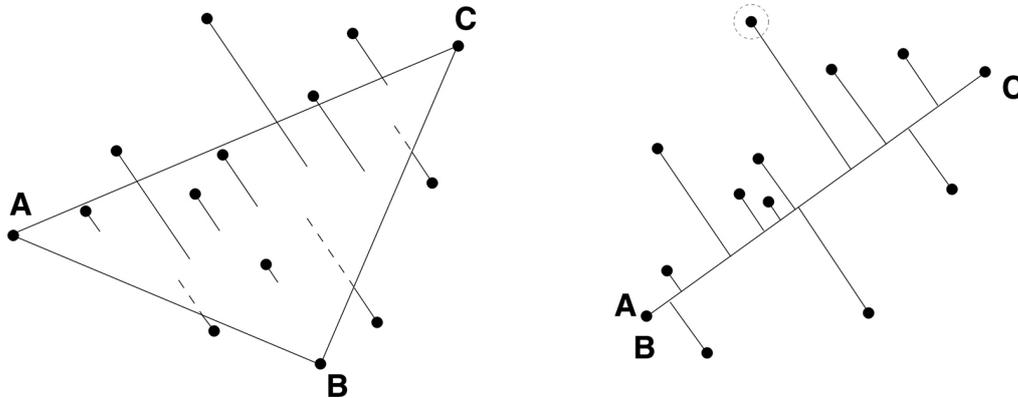


Figure 4. Point selection. The most distant point from the triangle plane was inserted into the triangulation, provided that this distance was greater than a specified threshold (figure from Midtbø [12])

2.4 Evaluation Criteria

These criteria were chosen to evaluate the approximation errors of the various methods: surface area, volume, root mean square (RMS) of vertical residuals¹, mean of vertical residuals and maximum vertical residuals. Surface area calculations and volume calculations obtained from the reduced datasets were compared with those obtained from the corresponding full datasets. Vertical residuals were calculated as the difference between the measured values of the control points and the corresponding interpolated values from the TIN surfaces (3). Omitted data points are used as control points. To avoid boundary effects, such as removing points which belong to the convex hull of the point set, a considerable larger area than the «clipping window» (i.e. the square area of 1 km x 1km) was subjected to data reduction. The buffer zone was 200 metres. Effect of data reduction on the selected criteria was evaluated only for the area inside the clipping window. The vast majority of all triangles were either completely outside or completely inside the square clipping window. These two groups were identified by integer tests of co-

ordinate extent, since the clipping windows are axis-aligned. The remaining triangles could possibly intersect with the clipping window. If they did intersect, the output could in principle range from triangles to heptagons (seven-sided polygons) depending on the actual configuration of the input triangle and the clipping window. The clipping of this group of triangles was handled by a «Sutherland-Hodgman» polygon clipping algorithm (see for example Hern and Baker [13]). Nevertheless, all triangles clipped by the square window will become convex polygons. The convex property makes it straightforward to split polygons into triangles and the same framework for calculations of evaluation parameters could be applied to all triangles. The TIN model was used to calculate all of the evaluation criteria. Surface area of each individual triangle was calculated by (1) and then summarized for all triangles to get the total surface area within the clipping window. The volume of a single tri-prism was calculated by (2) and the total volume was obtained by summarizing the volume of each individual tri-prism.

$$SA(\Delta) = \frac{1}{2} |\bar{a} \times \bar{b}|, \text{ where } \bar{a} \text{ and } \bar{b} \text{ are vectors that span the triangle facet.} \quad (1)$$

$$V(\Delta) = A(\Delta) \frac{(z_1 + z_2 + z_3)}{3}, \text{ where } A(\Delta) \text{ denotes the horizontal triangle area.} \quad (2)$$

1. In this paper the terms *residual* and *error* are both used to refer to deviation from the full model.

$$z(x, y) = \frac{-ax - by - d}{c}, \text{ where } a, b, c \text{ and } d \text{ are calculated from the enclosing triangle.} \quad (3)$$

3 Results and Discussion

Many circumstances will affect the construction of a TIN model and consequently the derived parameters. Examples of factors influencing the results include: selection of initial points for triangulation, handling of degenerated cases and order of point insertion. Limited testing has shown that some of these factors are inferior to others, but this issue was not addressed in this study. In this study, all factors not related to the test datasets themselves were held constant. Selection of evaluation criteria is highly application dependent. This study had assessment of slide hazard in mind. Volume calculations are of course essential to slide assessment and the ratio of surface area to projection area can serve as a simple measure of terrain roughness. The vertical residuals provide a more general measure of approximation quality. A more advanced interpolation technique such as Kriging would probably lead to better results with respect to interpolated height values, but the intention here was to evaluate how well the TIN surface itself approximates the topographic surface.

Figure 5 shows the average difference in surface area and the associated standard deviations. The hypotheses of interest regarding surface area were:

- H_0 : All methods have the same means within each level of reduction
 H_1 : At least one of the methods has, for at least one level of reduction, a different mean than the other methods

In order to test these hypotheses, they were split into hypotheses for pairs of methods. Thus the testing could be carried out using a number of paired Student's t-tests (t-tests). Two conditions are required for a paired t-test; 1) Each pair of observations is statistically independent from the other pair of observations 2) The differences between each pair of observations are approximately normally distributed. The first condition was assumed to be fulfilled since there was no over-

lap between the 32 test datasets. Histogram plots of the pair differences gave no indication of severe violations of the latter condition. These paired t-tests involved all possible pair combinations of the five methods, i.e. first RD is tested against PD (for each level of reduction) than RD is tested against PH and so forth. The sub-hypotheses are as follows:

- H_0 : $\mu_i = \mu_j$ (method i has the same mean as method j for reduction level k)
 H_1 : $\mu_i > \mu_j$ or $\mu_i < \mu_j$ (method i has a larger/smaller mean than method j for reduction level k)
 where $i \in \{1..5\}, j \in \{1..5\}, k \in \{1..5\}, i \neq j$

The calculated difference value for each pair determined whether it should be a «less than» or a «greater than» test, thus the H_1 hypotheses were one sided. The two latter hypotheses were tested separately for each level of reduction, i.e. for 10%, 5%, 2.5%, 1% and 0.5% of full dataset. The testing was carried out at a significance level of 5% and 31 degrees of freedom. All five methods had significantly different mean values for every level of reduction. The order of the mean values was PH > VH > PD > VD > RD as suggested by Figure 5. Moreover, an evident trend for all methods was that less data gave lower surface area. This was expected, since a decrease in number of surface points will cause some smoothing of the finer topography as well as a reduction in random noise, which also makes a significant contribution to surface area.

Figure 6 shows the average difference in volume and the associated standard deviations. Plots of volume residuals indicated that they were close to normally distributed, making the t-test applicable. The hypotheses to be tested were as follows:

- H_0 : $\mu_i = 0$ (volume calculations associated with method i give unbiased estimates)
 H_1 : $\mu_i \neq 0$ (volume calculations associated with method i give biased estimates)

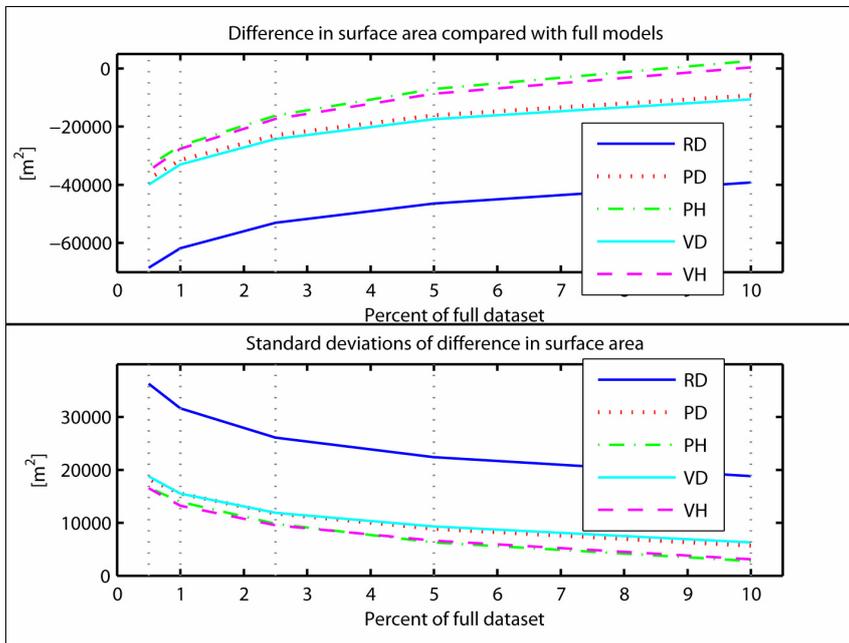


Figure 5. Surface area. Upper plot: average difference in surface area for the 32 test datasets. Lower plot: standard deviation of difference in surface area for the 32 test datasets. The vertical dotted lines indicate reduction levels applied in comparisons.

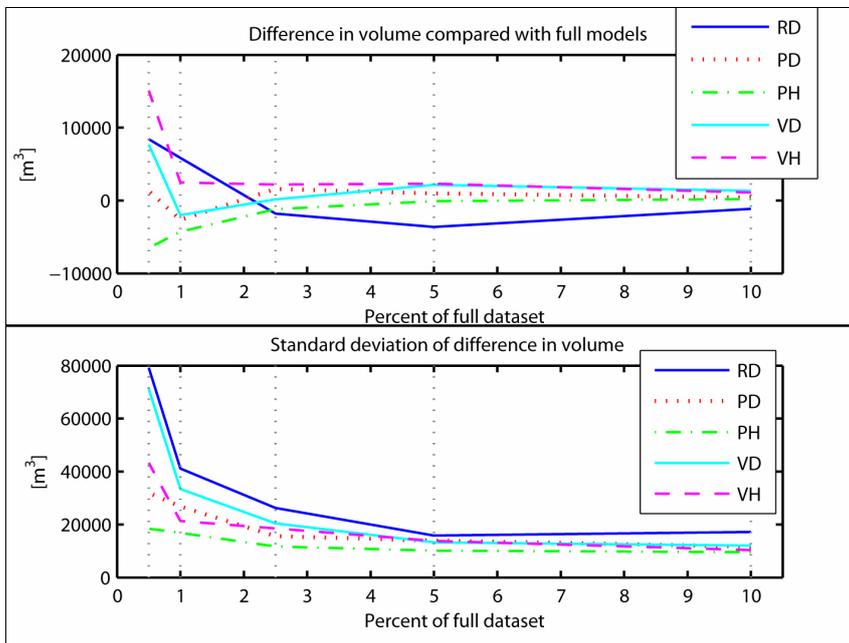


Figure 6. Volume. Upper plot: average volume difference compared with full models for the 32 test datasets. Lower plot: standard deviation of volume difference. Vertical dotted lines indicate reduction levels applied in comparisons.

Student's t-tests with significance level of 5% and 31 degrees of freedom were used for testing. Each level of reduction was tested separately. None of the H_0 hypotheses could be rejected at any level of reduction. This means that the volume calculations for with all five data reduction methods were unbiased for all levels of reduction.

Figure 7 shows the vertical RMS values associated with the five methods. The RMS was tested pair-by-pair (as for the surface area) using Fisher-tests (F-tests). In order to apply the F-tests, the RMS values were squared and thus assumed to be Chi-square distributed as implied by the F-test. The hypotheses were as follows:

- H_0 : $\sigma_i = \sigma_j$ (method i has the same RMS as method j for reduction level k)
 H_1 : $\sigma_i > \sigma_j$ or $\sigma_i < \sigma_j$ (method i has a larger/smaller RMS than method j for reduction level k)
 where $i \in \{1..5\}, j \in \{1..5\}, k \in \{1..5\}, i \neq j$

The number of degrees of freedom is virtually unlimited if one considers all 32 datasets as a single area. The total number of data points processed was about $2.6 \cdot 10^7$ where 0.5% to 10% were used to construct the TIN models. The remaining 90% to 99.5% were used as control points and thus corresponded to degrees of freedom. This was a pragmatic adaptation since the different datasets did not contain the same number of points. Due to the large number of degrees of freedom, all RMS values were significantly different at any reasonable significance level. For the reduction levels 0.5%, 1% and 2.5% the order of the RMS values was $RD > VD > VH > PD > PH$ where «>» means significantly greater. However, for reduction levels 5% and 10% this order was not valid. For those reduction levels, RD had the highest RMS and PH had the lowest RMS, but the order of the other methods was changed. Table 3 and Table 4 show comparisons of RMS for PD versus PH and VD versus VH, respectively.

Table 2. Vertical root mean square error of the control points for method PD and PH.

Percent of full dataset	10%	5%	2.5%	1%	0.5%
PD RMS [m]	0.222	0.303	0.411	0.621	0.864
PH RMS [m]	0.189	0.262	0.357	0.540	0.752
Percent reduction of RMS	15.0	13.6	12.9	12.9	12.9

Table 3. Vertical root mean square error of the control points for method VD and VH

Percent of full dataset	10%	5%	2.5%	1%	0.5%
VD RMS [m]	0.216	0.314	0.463	0.789	1.158
VH RMS [m]	0.194	0.284	0.419	0.710	1.047
Percent reduction of RMS	10.1	9.5	9.7	10.0	9.6

Hypotheses and testing procedures similar to those described for volume were applied to the mean of the vertical residuals. Residuals in this context refers to the deviation of elevation values of control points from those interpolated from the TIN surface. Hypotheses tests indicated that the means of residuals

were unbiased for all methods except the PH method. It was not expected that PH would be biased since the volume estimates based on PH are unbiased. This result might be related to the variable TIN resolution. Figure 8 shows mean values of the vertical residuals and the associated standard deviations.

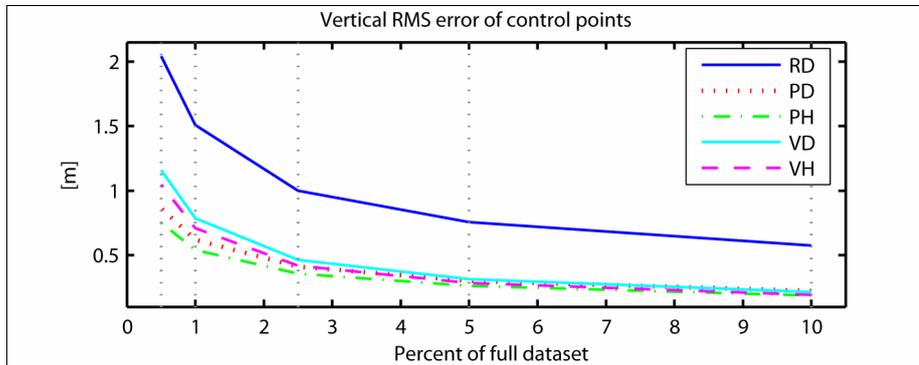


Figure 7. Vertical root mean square residuals of the control points. Residual values were calculated as interpolated values from the TIN surface minus measured elevation. Vertical dotted lines indicate reduction levels applied in comparisons.

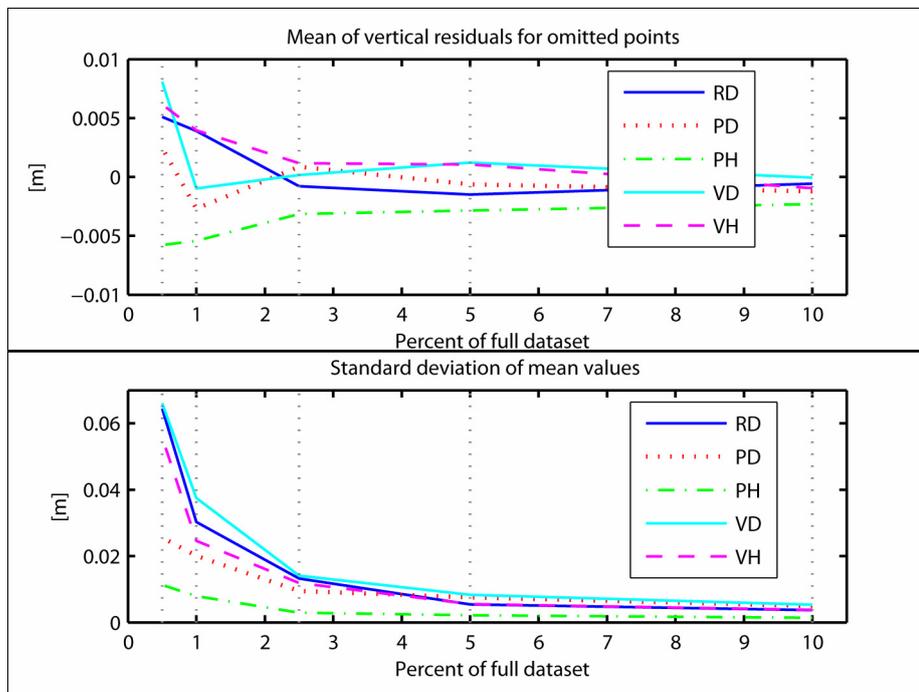


Figure 8. Mean of vertical residuals. Upper plot: mean values for the vertical residuals of control points. Lower plot: standard deviations of the mean values for the 32 test datasets. Vertical dotted lines indicate reduction levels applied in comparisons.

Figure 9 shows the average maximum vertical residuals and the associated standard deviation. As in testing of RMS, the maximum vertical residuals were squared in order to

apply F-testing. The same pairwise testing scheme were used as in the surface area tests. The hypotheses were:

- H_0 : $\hat{\sigma}_{\max,i} = \hat{\sigma}_{\max,j}$ (method i has the same mean of max residuals as method j for reduction level k)
- H_1 : $\hat{\sigma}_{\max,i} > \hat{\sigma}_{\max,j}$ or $\hat{\sigma}_{\max,i} < \hat{\sigma}_{\max,j}$ (method i has a larger or smaller mean of max residuals than method j for reduction level k)
 where $i \in \{1..5\}, j \in \{1..5\}, k \in \{1..5\}, i \neq j$

Again, the calculated mean of maximum residuals of the actual pair determined whether it should be a «less than» or a «greater than» test, making the H_1 hypotheses one-sided. A significance level of 5% and 31 de-

grees of freedom were used for testing. Testing showed that RD had a significantly larger mean of maximum residuals than the other methods. Moreover, PD and PH showed no significant difference in mean of maximum values. The same was true of VD and VH. Method PD and PH had a significantly larger mean of maximum residuals than VD and VH. The results from hypotheses testing regarding mean of maximum residuals can be summarized as follows: RD > (PD and PH) > (VD and VH), where «>» means significantly greater.

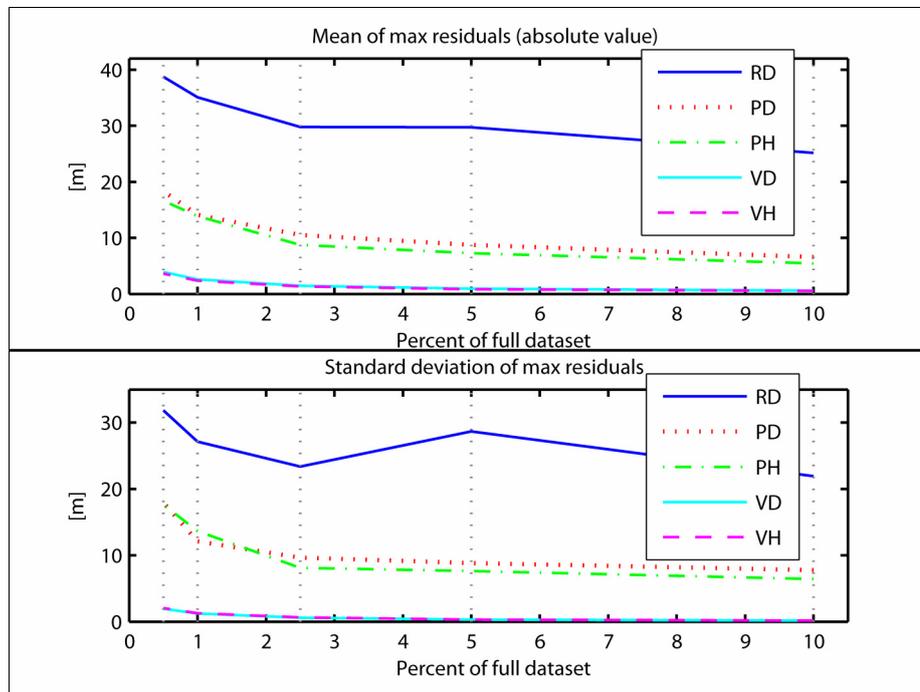


Figure 9. Max vertical residuals. Upper plot: means of the absolute value of the maximum vertical residual for the 32 test datasets. Lower plot: standard deviations of the maximum residuals. Vertical dotted lines indicate reduction levels applied in comparisons.

The calculated evaluation parameters are dependent on the type of terrain and the properties of the actual datasets. We found no indication that there were significant differences in relative performance (i.e. ranking of methods) among the five data reduction

methods when they were applied to different types of terrain. However, this might be due to poor categorization of terrain combined with a small number of test datasets.

Although the Delaunay criterion produces a «good quality» triangulation in terms of its

triangles' aspect ratios, it does not necessarily produce the triangulation which is the best approximation of a given height field, for two reasons: 1) sliver triangles are necessary to give a good approximation to some surfaces and 2) the swapping of edges which the Delaunay criterion invokes can cause artificial break lines where none exist in the original terrain [14]. A data dependent edge swap pass was performed for each of the two adaptive Delaunay triangulations. The processing cost associated with these edge swap passes was about 25% extra compared to the pure Delaunay triangulation. The gain of these data dependent edge swap passes was about 10-15 percent reduction of RMS error as well as improved approximation of surface area. In agreement with these finds, Garland and Heckbert [15] and Rippa [16] also reported improved results with a hybrid Delaunay – data dependent triangulation.

An ideal data reduction method should lead to unbiased results with low approximation errors as well as being inexpensive in terms of processing. Although the random subset method has some desirable properties, such as being unbiased for the parameters evaluated (except for surface area), the overall performance of this method is not adequate due to large variation in parameter estimates. This was expected, and the random method was introduced only as a reference for the naive approach. The PH method produced lower RMS than the PD method in all 32 test datasets as well as all 5 reduction levels. Similarly, the VH method produced lower RMS than the VD method for all test datasets and for all levels of reduction. Statistical testing of PD RMS versus PH RMS is unnecessary since the data dependent edge swap is a deterministic operation whose only function is to reduce the RMS. However there is a stochastic element involved since this is a greedy approach; during the edge swap process the same triangles can theoretically be subject to swapping several times and there is no guarantee that the optimal solution is found. The situation in VD and VH is slightly different. Although there is a deterministic operation to swap diagonals to minimize the largest residuals, this is not directly related to the RMS which is to be evaluated.

The PH method had the smallest RMS. It also produced the largest surface area and led to an unbiased volume estimate with the lowest standard deviation. Nevertheless, this method, along with PD and RD, has a major drawback: it provides no control of the upper vertical approximation error. If this is considered to be an important property, only method VD and VH remain, and of these two methods VH provides the best overall approximation quality. However, it might be advantageous to combine the vertical and perpendicular point selection criteria to get the best from both methods: slightly lower RMS and better approximation of surface area together with the ability to limit the maximum vertical approximation errors.

Some limitations of the methods applied here should be mentioned. Many conventional Geographical Information Systems (GIS) do not allow for dynamic triangulation and consequently these methods are not applicable for data reduction. Moreover, the methods described in this paper are vulnerable to gross errors as they tend to give preference to gross errors if such are present in the data. Also, testing a wider range of evaluation parameters including slope and curvature would provide useful additional information on the methods' suitability.

4 Summary and Conclusions

Five data reduction methods associated with dynamic TIN models were compared. All methods were applied to the same test datasets of LIDAR topographic data. The performance of the various reduction methods was evaluated by means of surface area, volume, RMS of vertical errors and maximum vertical errors. Although the test datasets have somewhat variable properties with respect to topography and point density, one can point out some general tendencies. Most of the graphs for the test parameters appear to have a deflection at 2.5% of full dataset. This gives an indication of a suitable upper level of reduction. No indications of significant change in *relative* performance of the five data reduction methods were found when applied to different types of terrain. However this might be due to poor categorization of

terrain combined with a small number of test datasets. Depending on the importance of the evaluation criteria, method VH was considered the most favourable. Method VH employs point selection by vertical threshold in dynamic Delaunay triangulation combined with a final data dependent edge swap pass to reduce the maximum vertical approximation errors. The final data dependent edge swap pass requires about 25% extra processing time with the reduction levels and implementation used in this study. The gain is a reduction of about 10-15% in RMS and improved approximation of surface area.

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